An ecological inference of statistics concerning size composition (I)

Ichiro YAMANAKA

§ 1. Preface

In the preceeding report$^{(1)}$, the author showed by an example, the statistics appearing in the estimation of size composition, c_1, c_2, and c_3, i. e, the variances within and between the days or trips drawn as sample, or $\rho(xjk, xik)$ the correlation coeficients between the number of individuals belonging to a certain size interval and those of the whole catch (in number) drawn as samples, are not only of mathematical value, but also have biological meanings as clues to know the structure of fish shoals.

In this report, the authr extended this method to the large sardines in the whole area in the coast of Japan Sea.

The meterials dealt with here were collected by prefectural fisheries experimental stations in the fisheries season of large sardines, namely in spring, of 1952.

§ 2. Estimating formulae.

In the survey of this year, at most fishing ports which were appointed as sampling stations, only one boat was drawn as sample in a day. Therefore it was impossible to seperate the inter-day variances from the intra-day ones, accordingly the following formulae were used in estimation.

The total number of trips at a station,	B
The trips number of drawn as samples at the same station,	b
The total number of fish caught by the k-th sample trip,	X_k
Number of fish drawn as samples from the above boat,	x_k
Numders in above, those beloging to the i-th size interval	
in total catch	X_{ik}
in sample	x_{ik}
The proportion of those being to the i–th interval in sample	

$$p_{ik}=\frac{X_{ik}}{X_k} \qquad q_{ik}=1-p_{ki}$$

……(1)

The simple extimate of the i-th interval can be got by

$$Y'_i=\frac{B}{b}\sum_k\frac{X_k}{x_k}x_{ik} \quad \cdots\cdots(2)$$

with the vaviance

$$V(Y_i)=B^2\frac{B-b}{B-1}\frac{\sigma_e^2}{b}+\frac{B}{b}\sum_k X_k^2\frac{X_k-x_k}{X_k-1}\frac{p_{ik}q_{ik}}{x_k}=V_1+V_2 \quad \cdots\cdots(3)$$

$$\left.\begin{aligned}\sigma^2{}_e &= \frac{1}{B}\sum_k (Y_k - Y)^2 \\ P'_i &= \frac{Y_i}{\Sigma Y_i} \\ C &= \frac{\sigma^2{}_e}{Y}\end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (4)$$

These formulae correspond with those in the preceeding report when the suffixes $_{1,\,2}$, which shaw the variances between boats within a day are eliminated, and on the other hand suffix $_1$ is attached to the variance between trips including both between and within days.

As to $\rho(N_k, N_{ki})$ no partcular caution is required comparing to the preceeding report.

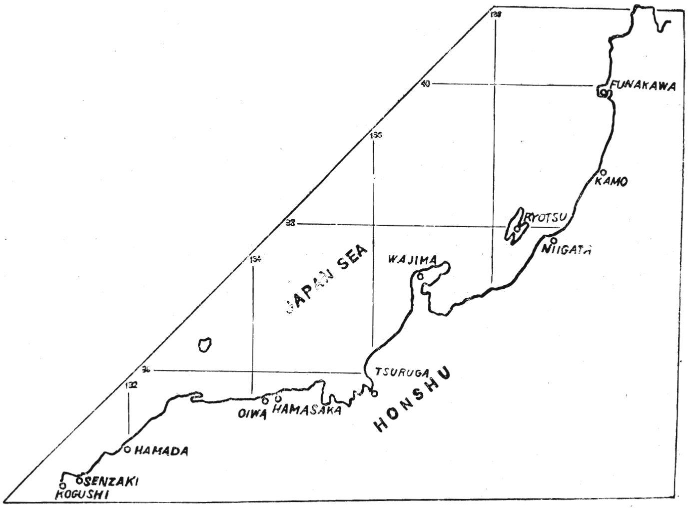

Fig. Locations of sampling stations

§ 3. Discussion on C, the variance coefficients of estimates.

In this report, the terms "size" and "magnitude" have different meanings, the former means body length and the latter does the bulk of shoals or catches.

The values of Cs show their minima 1.1–1.3 at the modes of size compositions and maxima at the tails of them, corresponding with geographical cline of the modes which have higher values in the northern countries, the values of Cs at the larger tail i. e. 21.5–2.20 c.m. groups remain small in this distrct.

It is noteworthy that these values are almost homogeneous throughout whole districts at 18–21.5 c. m. which can be considered to be the dominant groups.

As to Y', the estimated total number belonging to the i-th size interval, variance coefficents have also the homogenous values 1. 1–1. 4, nearly constant throughout the dominant size gorups.

The fact that the values of C has homogeneous values, suggests that the logarithms fof Ys distribute normally which can be assured approximately by using the

Table I. size composition in (%)

B. L. in C. m.	Kogushi	Senzaki	Hamada	OiWa	Hamasaka	Tsuruga	Wajima	Niigata	Ryotsu	Kamo	Funakawa
13.0			0.0								
13.5			0.0								
14.0			0.0								
14.5			0.0								
15.0			0.0								
15.5			0.0					0.1			
16.0			0.0				0.1	0.5		0.2	0.4
16.5	0.0	0.2	3.9	0.2	0.4		0.1	0.6		0.5	
17.0	0.1	0.9	2.0	1.1	0.4		0.2	1.6		1.9	
17.5	2.2	1.4	4.3	1.3	3.2	0.4	0.5	2.7		2.0	0.3
18.0	6.0	3.4	8.6	4.5	10.0	0.7	2.0	5.1		2.3	2.3
18.5	13.5	8.0	14.9	13.0	19.4	2.8	5.8	5.5		3.0	6.6
19.0	18.2	10.6	16.8	11.8	21.9	12.9	11.6	15.0		7.1	15.9
19.5	18.7	15.9	17.2	18.4	20.3	18.7	17.0	16.8	2.4	10.0	20.8
20.0	14.3	17.3	13.8	13.9	10.1	18.6	17.1	16.6	7.5	19.1	20.4
20.5	10.9	12.6	7.3	12.7	8.3	14.3	16.6	14.5	7.5	21.7	16.5
21.0	6.8	9.0	6.8	12.1	2.6	14.0	12.8	9.8	14.6	15.9	7.5
21.5	5.4	7.9	3.1	6.8	1.4	8.2	7.7	5.2	14.5	8.3	4.9
22.0	1.7	4.5	1.2	2.5	0.6	3.4	4.2	2.7	14.1	3.8	2.3
22.5	0.8	3.4	0.4	1.1		3.7	2.1	1.8	13.9	3.3	2.1
23.0	0.7	3.1		0.4	1.8	0.5	1.1	0.9	10.8	0.4	0.3
23.5		0.9	0.0			1.0	0.6	0.4		0.5	0.1
24.0	0.3	0.6				0.6	0.3	0.1			
24.5		0.2					0.0				
25.0											

probability-section papers.

It has been already shown by many authors[2] that the logarithms of catch per trip distributes normally, not only in the fisneries of large sardines by drift nets but also in the other kinds of fisheries,

Two facts are suggested biologically from these phenomena. The first of them is the inference of the fact that the logarithms of the catch per trip distribute normally. Assume that many causes determine the magnitude of catches, for example, the magnitudes of shoals themselves, the efficiency of gears, the skill of fishermen, the conditions of fishing and so on, and these factors are affected by many internal and external complicated factors, which operates stochastically, whose density are not required to make normal distribution. Put each of them Δz, then the summation $\Sigma\Delta z$, approximates the normal distribution by the centrum limiting theorum. And when each factor operates so as to effect in proportion to the magnitude at that time, in another saying, the magnitude of shoal to be Q, and $Q/Q\Delta = K\Delta Z$ then, $\log Q$ distributes normally. But we have to take care that this is only one of the possible inferences, but is not necessary to be the unique solution.

The second is, that, the shape of the distribution in dominant size groups of catch is nearly homogeneous throghout all stations, and the value of Cs are nearlly coincides each other. If, no correlation is found between each size group, and they are independent each other, then the value of C of the sum of each group should be less than that of each part, but this is denied by the fact.

From this fact, it is assumed that there are correlations between each size groups, and this fact is available as the clue to introduce the correlation coefficients $\rho(x_i, k)$ $\rho(X_{ik}, X_{ik})$ which show the construstion of shoals.

But as to the groups far from the dominant size groups, this phenomenon is not apparent.

Table II. $C=\frac{\sigma^2_e}{Y'}$

B. L.	Kogu-shi	Senza-ki	Hama-da	Oiwa	Hama-saka	Tsuru-ga	Wajima	Niigata	Ryotsu	Kamo	Funa-kawa
13.0			4.0								
13.5			2.9								
14.0			3.0								
14.5			2.9								
15.0			3.3								
15.5			3.4					2.8			
16.0			4.2				7.9	2.9		3.9	2.8
16.5	0.0	4.0	3.6	3.0	0.0	0.0	6.9	3.7		2.9	
17.0	0.0	5.0	2.1	2.3	0.0		2.3	3.2		3.5	
17.5	2.2	2.4	1.5	3.2	1.7	1.8	2.3	2.6		2.5	0.0
18.0	1.8	2.1	1.3	2.0	1-3	1.8	2.0	2.8		2.4	1.7
18.5	1.6	1.8	1.5	1.4	1.4	1.3	1.6	2.7		1.7	1.2
19.0	1.5	1.6	1.6	1.3	1.2	1.4	1.3	2.7		1.9	1.5
19.5	1.3	1.9	1.6	1.7	1.5	1.3	1.3	1.8	2.1	1.4	1.5
20.0	1.7	1.6	1.7	1.6	1.5	1.3	1.6	1.5	1.3	1.8	1.3
20.5	1.6	1.5	1.6	1.7	1.5	1.4	1.1	1.6	1.3	1.6	1.2
21.0	1.2	1.6	1.5	1.7	3.0	1.6	1.2	1.3	1.7	1.6	1.4
21.5	3.5	2.2	1.6	1.8	3.0	1.4	1.2	1.4	1.5	1.2	1.5
22.0	2.8	2.2	3.0	3.1	0.0	1.4	1.5	1.4	1.5	1.2	1.5
22.5	2.8	3.1	2.5	3.0		2.1	1.7	1.3	1.7	2.0	2.3
23.0	3.3	4.8		4.0	4.0	2.9	1.9	1.1	1.7	2.9	2.0
23.5		3.8				13.0	2.2	2.1	2.2	2.6	2.9
24.0	10.0	3.3				3.6	2.7				
24.5		4.5					4.7				
whole sizes	1.4	1.4	1.2	1.4	1.4	0.8	1.1	1.4	1.5	1.3	1.2
n	27	47	18	12	11	9	51	45	5	21	14

Table III. $r(X_{ik}, X_k)$ (Sample correlation coefficients)

B. L. in c.m.	Kogushi	Senzaki	Hamada	Oiwa	Hamasaka	Tsuruga	Wajima	Niigata	Ryotsu	Kamo	Funakawa
13.0			0.1								
13.5			0.5								
14.0			0.6								
14.5			0.6								
15.0			0.7								
15.5			0.6					0.2			− 0 2
16.0			0.7				0.1	− 0.1		0.3	
16.5			0.7	0.1			0.1	0.1		0.3	
17.0		0.4	0.6	0.4			0.0	0.6		0.3	
17.5	0.8	0·3	0.8	0.1	0.7	0.1	0.1	0.9		0.2	
18.0	0.8	0.3	0.7		0.9	0.3	0.8	0.9		0.3	0.9
18.5	1.0	0.6	0.6	0.8	1.0	0.5	0.7	1.0		0.7	0.9
19.0	0.9	0.6	0.6	0.9	1.0	0.6	0.8	1.0		0.5	1.0
19.5	0.9	0.7	0.6	0.9	1.0	0.9	0.9	1.0	0.1	0.9	1.0
20.0	0.9	0.7	0.6	1.0	0.9	1.0	1.0	1.0	0.8	1.0	1.0
20.5	0.8	0.9	0.6	0.8	0.9	0.9	0.8	0.8	0.8	0.9	0.9
21.0	0.9	0.8	0.5	0.8	0.8	0.9	0.9	0.8	1.0	0.9	0.9
21.5	0.8	0.6	0.3	0.7	0.8	0.9	0.7	0.7	1.0	0.9	0.4
22.0	0.8	0.7	− 0.1	0.3		0.9	0.7	− 0.4	1.0	0.9	0.7
22.5	0.8	0.6	0.4	0.3		0.9	0.8	0.3	1.0	0.9	0.9
23.0	0.6	0.7			0.6	0.8	0.6		1.0	0.3	0.0
23.5		0.6				− 0.8	0.7		1.0	0.4	0.1
24.0	− 0.1					− 0.1	0.3				
24.5							0.2				
							− 0.1				

§ 4. Discussion on $\rho(Xik, Xk)$

As mentioned above, these statistics show the degree of dependence of the number of fish belonging to a certian size group to the magnitude of the shoals, saying that ρ^2 times of the variance of number of any size groups is caused by that of the magnitudes of shoals.

At both Senzaki (Yamaguchi Pref.) and Hamada (Shimane Pref.) these value are, on the whole, low throughout the all range of size, the estimates of ρ^2 i, e, r^2 being below 0.5. But in other stations, these values are considerably high especially in dominant slze groups, The ratio estimates of size composition is the application of this fact.

In the size groups far from the dominant size, the value of the sample correration coefficients decrease in accordance to those of Cs, this seems as a commom phenomena in all stations, in coincidence with the geographical cline of the values of Cs, that is, the values of rs in the dominant size increase as in the northern countries.

The considerablly large values of r show that the variance of the number of a certain size group comes mostly from that of the magnitude of the shoal, in other words, the role of that group to the catch of that sample trip is, on the whole, homogeneous. On the other hand the low values at the both tails of size distribution show that the magnitude of these group does not coincide with those of the shoals to which these groups belong; in another saying, these ones extraordinally make aggregations indepently from the shoal of ordinally ones.

It is assumable that both at Senzaki and Hamada, the construction of shoals are not homogeneous, perhaps this can be explained by introducing two groups of migration, but we do not mention here the detail of it, as it is beyond of the aim of this report.

§ 5. On the discussion of the divergence coefficient F

Tabll IV. divergence coefficients $F=\frac{V(X_i)}{X_i}$

B. L.	Kogu-sui	Senzaki	Hama-da	Oiwa	Hama-saka	Tsuru-ga	Wajima	Niigata	Ryotsu	Kamo	Funa-kawa
13.0			0.7								
13.5			1.3								
14.0			3.0								
14.5			3.9								
15.0			4.8								
15.5			5.5								
16.0			9.7				0.6	3.1		0.2	0.1
16.5	0.0	0.2	4.5	0.1	0.0	0.0	1.0	3.1		0.3	
17.0	0.0	0.5	0.7	0.5	0.0		0.2	0.1		1.4	
17.5	1.1	0.4	0.6	0.9	0.1	0.1	0.4	0.6		0.7	
18.0	1.7	0.8	1.0	0.2	0.2	0.1	1.2	1.2		0.8	1.2
18.5	3.1	1.8	2.3	1.3	0.4	0.2	2.2	2.1		0.3	1.8
19.0	2.7	1.6	3.0	1.1	0.4	0.7	3.0	3.1		0.8	6.6
19.5	2.8	3.7	3.2	2.9	0.4	0.8	4.9	3.7	0.8	0.6	9 3
20.0	4.2	2.8	2.7	2.5	0.2	1.1	3.9	4.3	0.9	1.8	6.3
20.5	2.4	1.4	1.4	1.1	0.1	0.8	3.9	4.3	0.9	1.6	4.1
21.0	1.2	1.4	0.9	1.3	0.1	1.4	3.1	3.1	2.8	1.2	1.9
21.5	5.2	2.1	0.4	0.8	0.1	0.5	2.2	2.0	2.5	0.4	1.4
22.0	1.4	1.2	0.4	0.6	0.0	0.4	1.6	1.8	2.5	0.2	0.1
22.5	1.0	2.1	0.3	0.1		0.5	1.3	1.2	2.8	0.4	0.3
23.0	0.7	3.9			0.2	0.3	0.9	0.9	2.8	0.2	0.3
23.5		0.7	0.0			1.8	0.4	0.6	3.8	0.2	
24.0	0.3	0.3				0.1	0.3	0.4			
24.5		0.2					0.4				
whole sizes	19.4	14.0	15.0	19.5	1.7	5.3	20.4	1.4	16.8	5.7	21.0

As a method to know the construction of population from the shape of distribution, the divergence coefficients, i.e. the ratio of sample variance to the sample mean x were introduced.

It wae discussed by TORII[9] that, when this value exceeds 1, the distribution is of *concentrated shape*. and when it is below 1 it is of *exclusive shape*, while, when it is epuall to 1, it makes the *Poisson distribution* which shows that the distribution is quite at random as solitary individuals. But this mathod aims originally in a certain unit divisior, (in the population of insects, this unit is a netting) Therefore, it is natural that it should show a heavily concentrated type of distribution when applied to the catch of drift net, of which the scale of unit divisior is large.

As the dimension of nominators of Fs are different from the denominators, i.e. the former is of quadric from, while the latter of the first order, the value of F changes in accordance to the magnitude of the unit of individuals. Though many types of concentric distributions are considered, for example, *contagious*, *compound*, *convaluated* and othr types of distributions, a biological consideration is inevitable to infer the shapes of distributions. But here, in preceeding to these biological considerations, we made attention to the fact that the value of this F changes that of the magnitude of unit division, and considered, on the contrary, what value of unit should be taken in order to make the value of F to be one, that is to make the distribution to be stochasical, and dealtthese values of unit as "*Unit shoal*", In table V are shown the value of Fs, when at housand is dealt as the unit.

Put F the value of $\bar{F}$-distribution with the significant level α and of degree of freedom $n-1$, k, and-$\underline{F}$ is that with $1-\alpha$, ∞, $n-1$ respectablly, the upper and the lower limit of K the value on unit which should be taken in order to make the distribution of *Poisson* type will be $F/\bar{F}$, and $F/\underline{F}$ respectablly.

Anyhow, the values of Ks are also large at the dominant size groups, about 1.0–2.5 times of the average value of those groups, at the both tails of size compositions, though the value of those are small, the *ratios* of k to the average catch in those size groups is rather large compared with that dominant group If we consider the agregation of the magnitude to make the poisson distribution as the fisheries shoals, the density of these shoals decrease in the ends of the size distribution more rapidlly than the magnitude itself does. As to the whole catch of trip in ports with poor landing as Hamasaka, Tsuruga or Oiwa, the magnitudes of these unit shoals are between nine and fifteen thousands, while in the ports of rich landing such as Niigata, Hamada, or Senzaki, their values are between thirty and one hundred thousands. And in this case, the magnitudes of the unit shoals increase more remarkablly than those of the large individuals even though the average catch is less than that of the large sardines.

§ 6. The discussion on $\rho(X_{ik}, X_{i'k})$

As have been related, shoals have different habits in accordance with their size respectablly, but the individuals of each size do not exist quite independently but assemble into a synthetic structure, so having correlations between each size groups.

In this viewpoint the correlation coefficients between each size were tabulated,

Table V. The confidential intervals of the magnitudes of "unit shoal"

B. L.	Kogu-shi	Senzaki	Hama-saka	Hama-da	Oiwa	Tsuru-ga	wajima	Niigata	Ryotsu	Kamo	Funa-kawa
13.0			0.4 1.1								
13.5			0.8 2.1								
14.0			1.8 4.8								
14.5			2.9 6.2								
15.0			3.0 7.7								
15.5			3.4 8.9								
16.0			6.0 9.2					4.2 2.3		0.1 0.3	0.1 0.2
16 5		0.1 0.3	2.8 7.5		0.1 0.2		0.5 0.9	4.2 2.3		0.2 0.5	
17.0		0.4 0.7	0.4 1.1		0.3 1.2		0.7 1.3	1.4 0.7		0.9 2.6	
17.5	1.65 0.65	0.3 0.6	0.4 1.0	0.1 0.2	0.3 1.4	0.0 0.2	0.1 0.2	0.8 0.4		0.9 2.6	
18.0	1.1 2.5	0.6 1.1	0.7 1.6	0.1 0.6	0.1 0.4	0.0 0.2	0.3 0.5	1.6 0.9		0.5 1.3	0.1 2.7
18.5	2.1 4.6	1.4 2.5	1.4 3.7	0 2 1.1	0.7 3.1	0.1 0.5	1.6 3.1	2.9 1.5		0.2 0.6	1.1 4.0
19.0	1.8 4.1	1.2 2.3	1.8 4.8	0.2 0.9	0.6 2.6	0.4 2.0	2.2 4.2	4.2 2.3		0.5 1.5	4.0 14.6
19.5	1.9 4.2	2.6 5.2	2.0 5.4	0.2 1.1	1.6 7.0	0.4 2.0	3.6 6.9	5.0 2.7	0 3 4.4	0.4 1.1	5.8 20.6
20.0	2.8 6.3	2.0 3.9	1.7 4.5	0.1 0.6	1.3 6.0	0.6 3.2	2.9 5.5	5.8 3.1	0.4 4.9	1.1 3.3	3 9 13.9
20.5	1.6 3.6	1.0 2.0	0.9 2.3	0.1 0.3	0.6 2.6	0.4 2.3	2.9 5 5	5.8 3.1	0.4 4.9	2.1 2.9	2.5 9.1
21.0	0.8 2.1	1.0 2.0	0.5 1.4	0.1 0.3	0.7 3.1	0.7 4.1	2.4 4.3	4.2 2.3	1.2 15.7	0.8 2.2	1.6 5.7
21.5	3.5 8.8	1.5 2.9	0.3 0.7	0.1 0.3	0.4 1.9	0.3 1.5	1.6 3.1	2.7 1.5	1.1 14.1	0.5 0.8	0.6 2.1
22.0	0.9 7.9	0.8 1.7	0.2 0.6		0 3 1.4	0.2 1.2	1.2 2.2	2.4 1.3	1.1 14.1	0.1 0.4	0.6 2.0
22.5	0.6 1.6	1.5 2.9	0.2 0.4		0.1 0.2	0.2 1.3	1.0 1.8	1.6 0.9	1.2 15.7	0.2 0.7	1.1 3.8
23.0	4.7 1.3	2.8 5.5		0.1 0.5		0.1 0.7	0.7 1.3	1.2 0.7	1.2 15.7	0.1 0.3	0.2 0.6
23.5		0.5 0.8	0.0 0.0			0.9 5.2	0.3 0.6	0.8 0.4	1.7 15.7	0,1 0.4	0.1 0.5
24.0	0.2 0.5	0.2 0.4				0.1 0.4	0.3 0.4	0.5 0.3			
24.5		0.1 0.2					0.3 0.6				
25.0											
whole sizes	12.9 32.4	10.8 19.5	9.4 28.3	1.4 4.3	5.3 22.9	2.7 15.5	13.1 28.3	10.3 20.0	7.3 94.5	3.6 10.4	13.1 46.2

in Table (6). In strictlly saying, the partial correlations should be used, but the trouble of computation has hindered this work.

These correlation cofficients are significant during the range of about 1.5 c.m. (a range of a same year class) and these values are pretty high even in the ends of size distributions.

§ 7. Discussion on age classes

These discussions are available when applied in the age compositions and the correlation of each age groups.

To this purpose, the size compositions of each sample was converted into the age composition and computed by the same fromulae. The values of variance coefficients of the dominant age groups, two and three years groups, are low compared with those of other years, in northern districts where elder age groups are abundant, these values are nearly equall as in the case of size compositions, but slightly larger, perhaps due to the expanding of variance in the conversion from size to age composition.

The correlation coefficients between an age group and the total catch $\rho(Xik, Xk)$ are also high at dominant age groups, and low at old year groups, with an exception of the four years group at Kamo.

The divergence coefficients Fs and the magnitudes of the unit shoals have also the similar natures to the case of size composition. The magnitude of the unit shoal is largest in the three years group, and decreases rapidly in the elder age groups exept in the southern area.

For the correlation cefficients between the age groups, the high values are shown in the neighbouring ages even in the elder years of five or six years of age. But those values between every other ages, in generally, are significant only in northern districts, that is, a certain age group has influence in the southern area, to the year after next.

Thus, most discussion on the size composition are available in the age composition, but the accuracy decreases as the estimation errors have been introduced at the coinversion from size to age compositions.

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Table VI. Correlation coefficients between diferent size interval

1) 0.5 c.m. interval

	Senzaki	Hamada	Tsuruga	Wajima	Kamo
13.0-13.5		0.62			
13.5-14.0		0.92			
14.0-14.5		0.98			
14.5-15.0		0.95			
15.0-15.5		1.00			
15.5-16.0		0.98			
16.0-16.5		0.98		0.98	0.86
16.5-17.0	0.18	0.89		0.71	0.93
17.0-17.5	0.52	0.72		0.39	0.92
17.5-18.0	0.84	0.70	0.6	0.36	0.93
18.0-18.5	0.69	0.95	0.67	0.81	0.42
18.5-19.0	0.83	0.93	0.99	0.91	0.33
19.0-19.5	0.69	0.92	0.96	0.93	0.95
19.5-20.0	0.72	0.98	0.78	0.90	0.89
20.0-20.5	0.92	0.99	0.92	0.90	0.96
20.5-21.0	0.88	0.87	0.87	0.91	0.96
21.0-21.5	0.92	0.92	0.87	0.83	0.92
21.5-22.0	0.91	0.47	0.99	0.91	0.85
22.0-22.5	0.88	0.04	0.99	0.86	0.75
22.5-23.0	0.92		0.94	0.62	0.23
23.0-23.5	0.93			0.42	0.17
23.5-24.0	0.65			0.65	
24.0-24.5	0.17			0.14	

2) 1.0 c.m. interval

	Senzaki	Hamada	Tsuruga	Wajima	Kamo
13.0-14.0		0.53			
13.5-14.5		0.31			
14.0-15.0		0.96			
14.5-15.5		0·92			
15.0-16.0		0.96			
15.5-16.5		0.99			
16.0-17.0		0.87		0.74	0.13
16.5-17.5	0.53	0.73		0.14	0.96
17.0-18.0	0.73	0.34		0.19	0.93
17.5-18.5	0.53	0.44	0.64	0.38	0.47
18.0-19.0	0.50	0.90	0.79	0.76	0.13
18.5-19.5	0.75	0.93	0.84	0.84	0.56
19.0-20.0	0.71	0.86	0.63	0.81	0.80
19.5-20.5	0.59	0.97	0.59	0.87	0.88
20.0-21.0	0.69	0.92	0.89	0.84	0.93
20.5-21.5	0.67	0.80	0.77	0.74	0.90
21.0-22.0	0.91	0.46	0.92	0.88	0.78
2.15-22.5	0.68	0.53	0.98	0.82	0.91
22·0-23.0	0.71		0.95	0.90	0.15
22.5-23.5	0.97		0.83	0.94	−0.04
23.0-24.0	0.81			0.33	
23.5-24.5	0.18			0.53	

3) 1.5 c.m. interval

	Senzaki	Hamada	Tsuruga	Wajima	Kamo
13.0-14.5		0.87			
13.5-15.0		0.91			
14.0-15.5		0.92			
14.5-16.0		0.80			
15.0-16.5		0.94			
15.5-17.0		0.87			
16.0-17.5		0.68		0.14	0.95
16.0-10.0	0.56	0.16		0.22	0.99
17.0-18.5	0.64	0.99		0.17	0.38
17.5-19.0	0.39	0.44	0.64	0.25	−0.07
18.0-19.5	0.46	0.81	0.72	0.78	−0.99
18.5-20.0	0.66	0.86	0.62	0.70	0.47
19.0-20.5	0.53	0.85	0.64	0.76	0.74
19.5-21.0	0.23	0.83	0.58	0.67	0.78
20.0-21.5	0.45	0.90	0.74	0.66	0.86
20.5-22.0	0.73	0.02	0.82	0.77	0.70
21.0-22.5	0.79	0.94	0.92	0.78	0.70
21.5-23.0	0.53		0.81	0.83	0.68
22.0-23.5	0.34		0.89	0.56	0.52
22.5-24.0	0.74		0.17	0.53	
23.0-24.5	0.11			−0.03	

4) 2.0 c.m. interval

	Senzaki	Hamada	Tsuruga	Wajima	Kamo
13.0-15.0		0.29			
13.5-15.5		0.86			
14.0-16.0		0,85			
14.5-16.3		0.46			
15.0-17.0		0.82			
15.5-17.5		0.63			
16.0-18.0		0.13		0.23	0.85
16.5-18.5		−0.12		0.20	0.53
17.0-19.0	0.08	0.12		0.15	−0.04
17.5-19.5	0.33	0.28	0.39	0.21	−0.13
18.0-20.0	0.25	0.76	0.27	0.80	−0.16
18.5-20.5	0.40	0.84	0.71	0.62	0.51
19.0-21.0	0.33	0.69	0.29	0.53	0.65
19.5-21.5	0.17	0.84	0.52	0.40	0.73
20.0-22.0	0.53	0.18	0.84	0.63	0.65
20.5022.5	0.97	0.76	0.82	0.61	0.96
21.0-23.0	0.68		0.85	0.88	0.07
21.5-23.5	0.76		0.86	0.46	0.26
22.0-24.0	0.81		−0.05	0.52	
22.5-24.5				0.01	

5) 2.5 c.m. interval

	Senza-ki	Hama-da	Tsuru-ga	waji-ma	Kamo
13.0-15.5		0.21			
13.5-16.0		0.75			
14.0-16.5		0.84			
14.5-17.0		0.69			
15.0-17.5		0.60			
15.5-18.0		0.06			
16.0-18.5		−0.003		0.18	0.29
16.5-19.5	0.07	−0.11		0.18	−0.04
17.0-19.5	0.53	0.07	−0.05	−0.01	−0.09
18.0-20.5	0.19	0.20		0.15	−0.10
18.5-21.0	0.16	0.76	0.20	0.68	−0.10
19.0-21.5	0.12	0.69	0.21	0.42	0.60
19.5-22.0	0.11	0.82	0.20	0.26	0.58
20.0-22.5	0.22	0.06	0.56	0.52	0.60
20.5-23.0	0.57	0.76	0.82	0.45	0.99
21.0-23.5	0.68		0.84	0.78	0.25
21.0-23.5	0.81		0.88	0.39	0.17
21.5-24.0	0.57		0.18	0.41	
22.0-24.5	0.22			0.12	

6) 3.0 c.m. interval

	Senza-ki	Hama-da	Tsuru-ga	Waji-ma	Kamo
13.0-16.0		−0.01			
13.5-16.5		0.75			
14.0-17.0		0.72			
14.5-17.5		0.45			
15.0-18.0		0.05			
15.5-18.5		−0.20			
16.0-19.0		−0.16		0.17	−0.03
16.5-19.5	−0.04	−0.13		0.02	−0.13
17.0-20.0	0.34	0.04		−0.01	−0.06
17.5-20.5	0.16	0.17	0.13	0.09	0.10
18.0-21.0	−0.10	0.58	−0.12	0.54	0.02
18.5-21.5	−0.01	0.35	0.06	0.10	0.59
19.0-22.0	0.22	−0.09	0.14	0.38	0.46
19.5-22.5	0.04	0.65	0.59	0.30	−0.06
20.0-23.0	0.53		0.73	0.66	0.38
20.5-23.5	0.66		0.77	0.26	−0.01
21.0-24.0	0.71		−0.10	0.29	
21.5-24.5	0.11			0.02	

7) 3.5 c.m. interval

	Senza-ki	Hama-da	Tsuru-ga	Waji-ma	Kamo
13.5-17.0		0.62			
14.0-17.5		0.48			
14.5-18.0		− 0.03			
15.0-18.5		− 0.21			
15.5-19.0		0.14			
16.0-19.5		−0.16		0.03	−0.06
16.5-20.0	−0.05	−0.12		0.02	−0.09
17.0-20.5	0.34	0.03		−0.06	−0.03
17.5-21.0	−0.06	−0.07	−0.29	0.01	0.12
18.0-21.5	−0.10	0.67	−0.18	0.25	0.27
18.5-22.0	−0.01	−0.06	0.03	0.25	0.74
19.0-22.5	0.35	−0.32	0.20	0.16	0.82
19.5-23.0	0.20		0.47	0.47	0.50
20.0-23.5	0.52		0.82	0.15	0.05
20.5-24.0	0.76		0.04	0.11	
21.0-24.5	0.01			−0.08	

Table VII. X_i in age composition

(in 10^3)

age	Senza-ki	Oiwa	Tsur-uga	Waji-ma	Niiga-ta	Kamo
1					0.07	
2	2.26	2.5	0.6	4.6	5.8	0.7
3	3.0	1.9	2.1	11.2	8.3	3.2
4	0.6	0.1	0.4	2.4	2.2	0.3
5	0.1	0.04	0.1	0.3	0·5	0.1
6				0.1	0.1	
7				0.1	0.01	
8					0.01	

Table VIII. *C* in age composition

age	Senzaki	Oiwa	Tsuruga	Wajima	Niigata	Kamo
1					2.80	
2	1.40	1.50	1.25	1.18	1.10	1.80
3	1.33	1.60	1.30	1.12	1.20	1.70
4	1.70	2.20	1.60	1.16	0.91	1.45
5	2.40	2.70	1.90	2.20	0.95	2.10
6				1.70	1.20	
7				4.44	4.30	

Table IX. *F* in age composition

age	Senzaki	Oiwa	Tsuruga	Wajima	Niigata	Kamo
1					0.5	
2	9.8	5.7	0.9	6.5	7.5	2.2
3	15.2	5.0	3.4	12.2	11.6	9.0
4	17.3	0.8	1.1	3.2	1.8	0.7
5	0.1	0.3	0.2	1.5	0.4	0.2
6				0.3	0.2	
7				0.1	0.2	

Table X. $r(X_i, X_{ik})$ in age composition

age	Senzaki	Oiwa	Tsuruga	Wajima	Njigata	Kamo
1						
2	0.8	0.9	0.8	0.8	0.1	0.6
3	0.9	0.9	1.0	0.9	0.7	0.6
4	0.9	0.6	0.9	0.8	0.7	0.9
5	0.3	0.3	0.9	0.7	0.7	0.3
6	0.5			0.5	0.6	
7				0.0	0.3	
8				0.3		

Table XI. inter-age correlation
$r(X_{ik}, X_{i'k})$ in age composition

age	Senzaki	Oiwa	Tsuruga	Wajima	Niigata	Kamo
1–2					0.4	
2–3	0.8	0.8	0.8	0.9	0.8	0.1
3–4	0.8	0.7	0.9	0.9	0.9	0.5
4–5	0.04	0.9	0.9	0.9	0.9	0.3
5–6	−0.01			0.9	0.7	
6–7				0.5	0.4	
1–3					−0.02	
2–4	0.11	0.13	0.14	0.16	0.7	0.4
3–5	0.3	0.5	0.9	0.8	0.8	0.4
4–6	0,8			0.8	0.6	
5–7				0.2	0.4	
1–4					−0.05	
2–5	0.4	−0.02	0.3	0.4	0.7	0.6
3–6	0.5			0.5	0.4	0.4
4–7				0.1	0.0	
2–6	0.2					